

Heat Flow Through Conical Constrictions

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Abstract

IN many instances each asperity in a joint formed by two surfaces can be considered to be a frustum of a cone surrounded by either a vacuum or a conducting medium. This paper deals with the numerical solution of the associated heat-conduction problem and discusses the results in the light of previous work on constriction resistance.

Contents

In the study of actual area between contacting surfaces, it is often justifiably considered that the asperities in contact are conical in shape.¹⁻⁴ For blasted surfaces, for example, the mean of the absolute slope of profile is 0.1 (Ref. 5) implying that, in this case, the asperities in contact may be taken to be the frusta of cones with a semiangle of 80-85 deg. Also in many contact heat-transfer experiments, the contact configuration is of the crossed-wedge type or pyramids contacting flats.^{6,7} In these instances, it is more reasonable to idealize *one side of an individual contact spot* by a frustum of a cone fed by a cylinder. As shown in Fig. 1 this conical constriction may be surrounded by either a vacuum or a conducting medium.

This paper presents the results of the finite difference solution of the heat-conduction problem depicted in Fig. 1. The results are compared with previous analog and disk constriction results wherever such direct comparison is feasible. The results for the single contact spot may be applied to practical contact geometries in which the number and average size of contact spots can be determined from statistical and deformation considerations.¹⁻⁴

In each case, the temperature profile obtained from the numerical solution is examined to confirm that the solution indeed represents the physical situations shown in Fig. 1. One such profile is plotted in Fig. 2. A typical error introduced in the finite difference, analog, or indeed any experimental solution of the present problem is due to the axial length of the cylinder being necessarily finite. In the finite difference solution, an examination of the temperature profile reveals whether the total axial length considered is sufficient in relation to the radius of the cylinder and the extent of the conical portion.

The constriction resistance may be defined as the ratio of additional temperature drop ΔT (due to the presence of constriction) to the temperature gradient in the undisturbed field. The resistance is nondimensionalized by dividing it by the radius b of the feeding cylinder, i.e.,

$$R = \Delta T / b(dT/dz)_{\infty}$$

The constriction resistance is sometimes defined as⁸

$$R_c = f/4ak$$

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where f is the constriction alleviation factor and k the thermal conductivity of solid. Unlike this latter definition, the present definition is based as heat flow *per unit area* and is independent of k . The relation between the two is

$$R = \pi b k \cdot R_c$$

Figure 3 shows the variation of resistance with radius ratios. In this diagram C is the ratio of fluid conductivity to solid conductivity. The value of 0.005 for C is commonly obtained in practice, e.g., stainless steel contacts in air. The reduction in resistance due to the presence of fluid is

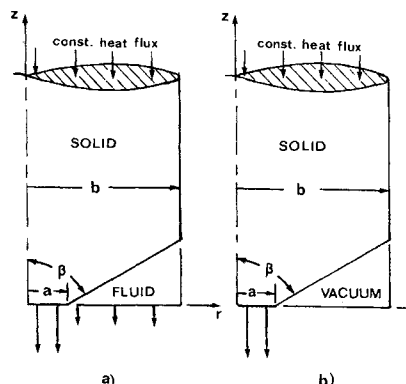


Fig. 1 Conical constriction: a) in fluid, b) in vacuum.

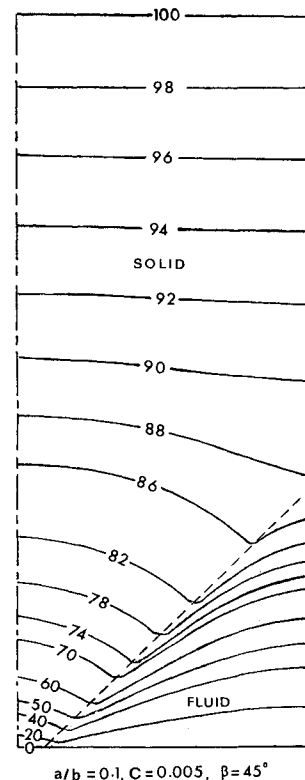


Fig. 2 Temperature contours.

$a/b = 0.1, C = 0.005, \beta = 45^\circ$

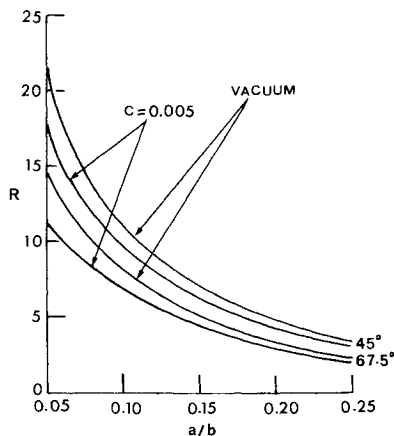


Fig. 3 Conical constriction: resistance vs radius ratio.

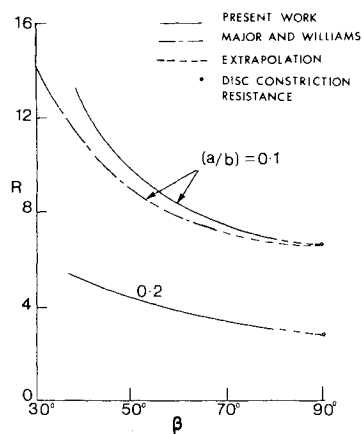


Fig. 4 Conical constriction in vacuum: resistance vs cone semi angle.

significant especially at the low radius ratios commonly obtained in practice.

It can be seen from Fig. 4 that the resistance decreases rapidly when the cone semiangle β is small, but that such reduction is less marked at larger values of β . The (non-dimensionalized) disk constriction resistances correspond to $\beta = 90$ deg. For β greater than about 80 deg, there seems to be no great difference between disk and conical constrictions in vacuum. In fact, at 80 deg the difference is about 2% and at 85 deg the difference is less than 0.5%. Therefore we can conclude that the disk constriction resistances could be used with confidence in the analysis of heat flow in a vacuum across joints formed by nominally flat rough surfaces for which β is about 80 deg.

In Fig. 4 are also shown the results of electrolytic tank analog test results of Ref. 9 for a radius ratio of 0.1. The analog simulates conditions in vacuum. Some discrepancy can be seen to exist between numerical and analog results, especially for small values of β . It is at these small values of β that the length of the cylinder is critical. In the finite-difference method, the temperature profile (Fig. 2) can be examined and the length increased if necessary. Such verification is difficult with the analog method.

Conclusions

Temperature profiles have been obtained for problems in heat flow through conical constrictions. The results of the analysis show that:

- 1) Constriction resistance is reduced significantly due to the presence of conducting fluid, especially at the practically important low radius ratios.
- 2) Although the resistance reduces with increasing cone semiangle, such reduction is less marked at larger semiangles.
- 3) The disk constriction resistances can be used in the analysis of heat flow across joints formed by nominally flat rough surfaces in vacuum.

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